Section 2.3

The Basic Limit Laws

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(1) The Limit Laws
(2) Examples
(3) Assumptions Matter

Basic Limit Laws:

Assume that $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ each **exist**.

Identity and Constant Laws $\quad \lim_{x\to c} x = c \qquad \lim_{x\to c} 1 = 1$

Sum Law $\quad \lim_{x\to c} (f(x)+g(x)) = \left(\lim_{x\to c} f(x)\right) + \left(\lim_{x\to c} g(x)\right)$

Constant Multiple Law $\quad \lim_{x\to c} (kf(x)) = k\left(\lim_{x\to c} f(x)\right)$

Product Law $\quad \lim_{x\to c} (f(x)g(x)) = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)$

Quotient Law

If $\lim_{x\to c} g(x) \neq 0$,

$$\lim_{x\to c} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

Power Law

If n is an integer,

$$\lim_{x\to c} (f(x)^n) = \left(\lim_{x\to c} f(x)\right)^n$$

(I) $\lim_{x\to c} x^3+4x^2-3$ (II) $\lim_{x\to c} \dfrac{x^4+x^2-1}{x^2+5}$ (III) $\lim_{x\to c} \dfrac{x^2-2x-15}{x^2-9}$

Example IV

$$\lim_{x \to -1} f(x) = 3 \qquad \lim_{x \to -1} g(x) = -2$$

$$\lim_{x \to 2} f(x) = -1 \qquad \lim_{x \to 2} g(x) = 4$$

With the above limit information, evaluate the limits:

(i) $\displaystyle \lim_{x \to -1} (2f(x) - 3g(x))$

(ii) $\displaystyle \lim_{x \to 2} \frac{x\sqrt{g(x)}}{f(x)^2}$

(iii) $\displaystyle \lim_{x \to -1} \frac{g(-2x)}{x^2}$

Assumptions Matter

Every Basic Limit Law rests upon the assumption that the **limits exist**!
If either limit fails to exist, the limit laws cannot be applied:

(Example V) The Product Law cannot be applied to $\lim_{x \to 0} f(x)g(x)$ if $f(x) = x$ and $g(x) = x^{-1}$.

Assumptions Matter

Every Basic Limit Law rests upon the assumption that the limits **exist**! If either limit fails to exist, the limit laws cannot be applied. However, the combination of limits may exist anyway!

(Example VI) Give an example where $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)}$ exists, but neither $\lim\limits_{x \to c} f(x)$ nor $\lim\limits_{x \to c} g(x)$ exist.